Current integration force and displacement self-sensing method for cantilevered piezoelectric actuators

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This paper presents a new method of self-sensing both of the displacement and the external applied force at the tip of piezoelectric cantilevers. Integrated electric current across piezoelectric actuators is compensated against material nonlinearities (creep, hysteresis) to provide reliable information. We propose to compensate the hysteresis by using the Prandtl–Ishlinskii static approach while an auto regressive and moving average exogenous (ARMAX) model is used to minimize the creep influence. The quasistatic estimation, electronic circuit, and aspects related to long-term charge preserving are described or referenced. As an experiment, we tested the actuator entering in contact with a fixed force sensor. An input signal of 20 V peak-to-peak (10% of maximum range) led to force self-sensing errors inferior to ±8%. A final discussion about method accuracy and its limitations is made.1

I. INTRODUCTION

Our previous work1 discussed self-sensing of free cantilever displacement. In this paper, we introduce a new algorithm based on the same electronic schematic for evaluating unknown applied force at the tip of the actuated cantilevers. Related to1 both the displacement and the applied force at the tip of the cantilevers are now observed, with the cost of a supplementary compensation of hysteresis and creep effects. The proposed approach can be adapted for long-term duration, up to several tens of seconds.

II. DISPLACEMENT AND FORCE DETECTION

We employ a cantilever of length $L$, width $w$, and thickness $h$. The setup is rather similar to the one already detailed in Ref. 1. Reference force was measured with a FT-S270 micromachined capacitive sensor from FEMTO-tools company mounted on a XYZ microtranslation table for close-contact adjustment. Reference displacement was provided by a Keyence LC-2420 optical device. Both sensors were employed as reference in identification and error evaluation tests; their presence is not needed for self-sensing method.

A. Equation of the estimate force

Theoretical charge due to applied voltage $V_{in}$ and external force $F_{ext}$ is linear

$$Q = \int \int_A \sigma dx_1 dx_2 = \int_0^L \int_0^w \left[ -e_{31} \frac{h}{2} \frac{12F_{ext}}{Ywh} (L - x_1) ight. \
+ 4e_{33}V_{in} \left. \frac{1}{h} dx_3 dx_1 \right] = -3e_{31}s_{11} \frac{L^2}{h^2} F_{ext} + \frac{4Lw e_{33}V_{in}}{h} \beta F_{ext} + C_P V_{in},$$

where $\beta$ is the force sensitivity coefficient and $C_P$ is the actuator capacitance. $C_R$ is an optional reference capacitor.1

Displacement of the beam submitted to external voltage and force can be derived from

$$\delta = \frac{4s_{11}^E L^3}{4s_{11}^E \rho s_{33}} \frac{L^3}{wh^3} F_{ext} - \frac{3d_{31}}{h V_{in}} - \frac{3}{1 + \frac{d_{31}}{2} s_{11}^E L^2 h^2 F_{ext} + \frac{4Lw e_{33}V_{in}}{h}}$$

If we add compensation against op-amp bias current $i_{BIAS}$ and piezoelectric actuator leaking resistance $R_{EP}$ (see Ref. 1), we get the following estimated external force:

FIG. 1. Force and displacement detection model implemented under MATLAB SIMULINK.

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The above expression does not take into account nonlinear nature of piezoelectric ceramics that introduce large parts of uncertainty. For instance, measurements on a unimorph beam of $15 \times 1 \times 0.2 \, \text{mm}^3$ showed $C_p = 1.74 \, \text{nC/V}$ and $\beta = 1.03 \, \text{nC/mN}$. Blocking force of such a beam was only $0.07 \, \text{mN/V}$. This indicates that 1% of error (nonlinearity) in charge-to-applied voltage characteristic introduces 24% uncertainty in the estimation of force. Given that ferroelectric behavior of the PZT material shows even 15% of nonlinearity, a compensation of these unwanted effects is unavoidable.

Hence, we will replace $C_p$ from Eq. (3) with more appropriate estimators. Hence, final expression of the estimate is taken from Ref. 1 if more accuracy is required. We added a supplementary third-order low-pass Butterworth filter was introduced to cancel the noise of $F_{\text{est}}/k_Z$ term from Eq. (6) which is far superior to $\delta_{\text{free est}}$.

C. Displacement and force estimator

The Simulink detection model of force and displacement implemented into a dSPACE real time controller sums several terms from Eqs. (4)–(6) and is shown in Fig. 1. As seen, a supplementary third-order low-pass Butterworth filter was introduced to cancel the noise of $F_{\text{est}}/k_Z$ term from Eq. (6) which is far superior to $\delta_{\text{free est}}$.

III. SELF-SENSING PARAMETER IDENTIFICATION

Identification for only displacement self-sensing method was previously discussed in Ref. 1. These parameters were bias current $i_{\text{BIAS}}$, leaking resistance $R_{FP}$, and displacement coefficient $\alpha$. For simplicity reasons, we suppressed dielectric absorption compensation ($Q_{DA}$) but it can be reintroduced from Ref. 1 if more accuracy is required. We added a supplementary series of parameters intended for force self-sensing, such as force sensitivity $\beta$ and transverse stiffness $k_Z$ identifiable from an external force step ($F_{\text{est}} \neq 0$, $V_{\text{in}} = 0$). Also hysteresis operator $F_H$ and creep transfer function $F_C$ have to be identified. These nonlinearities characterize the piezoelectric actuator behavior between applied voltage $V_{\text{in}}$ and free actuator bending $\delta$.

To compensate the hysteresis, we use the Prandtl–Ishlinskii (PI) approach because of its accuracy and ease of implementation and computation (Ref. 3). The compensation is performed by putting in parallel the system and the PI model (Fig. 2). In the PI model, a hysteresis is based on the play operator, also called backlash operator. A play operator of unity slope is defined by

$$V_{\text{out}}(t) = \max\{V_{\text{in}}(t) - r, \min\{V_{\text{in}}(t) + r, V_{\text{out}}^*(t - T)\}\},$$

where $V_{\text{out}}^*$ is the output voltage compensated against bias and leaking currents and summing $-C_R V_{\text{in}}/C$ reference capacitor term. Parameter $r$ is the threshold and $T$ is the sampling period.

A hysteresis can be approximated by the sum of several play operators weighted by the gain (slope) $w_i$. Let $n$ be the number of elements, so we have

$$V_{\text{out}}^*(t) = F_H(V_{\text{in}}) = \sum_{i=1}^{n} w_i \max\{V_{\text{in}}(t) - r_i, \min\{V_{\text{in}}(t) - r_i, V_{\text{out}}^*(t - T)\}\},$$

where $w_i$ and $r_i$ are the weight and transverse stiffness of the beam.

C. Equation of the estimate displacement

Formula for free ($F_{\text{est}} = 0$) piezoelectric beam displacement is taken from 1

$$\delta_{\text{free est}} = -\frac{C}{\alpha} V_{\text{out}} + \frac{C_R}{\alpha} V_{\text{in}} - \frac{1}{R_{FP}\alpha} \int V_{\text{in}}(t) dt - \frac{1}{\alpha} \int i_{\text{BIAS}}(t) dt,$$

where $\alpha$ is called the displacement coefficient.

When submitted to both external voltage and force, theoretical expression [Eq. (2)] is prone to nonlinearities, it is better to estimate the displacement with the following formula:

$$\delta_{\text{est}} = \delta_{\text{free est}} - F_{\text{est}}/k_Z,$$

which is far superior to $\delta_{\text{free est}}$.
bias, hysteresis) and, afterwards, the model $F_v(V_{in})$ is identified using the auto regressive and moving average exogenous (ARMAX) method and Matlab. It seems that from the third order, the error between the identified model and the experimental curve stops decreasing exponentially. We chose a model of fourth order

$$F_v(s) = \frac{V_{creep}(s)}{V_{in}(s)} = \frac{a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4}{s^4 + b_1s^3 + b_2s^2 + b_3s + b_4},$$

where $a_0, \ldots, a_4$ and $b_1, \ldots, b_4$ are numerator and denominator polynomial coefficients of creep transfer function.

**IV. RESULTS AND DISCUSSION**

The unimorph PZT on Ni cantilevered actuator (15 $\times$ 1 $\times$ 0.28 mm$^3$) is brought near the force sensor (close to contact). Then, a series of periodic steps [Fig. 4(a)] ranging from 0 to $-10$ or $-20$ V is applied, making the actuator entering in contact and pushing the sensor. The recorded output $V_{out}$ is pictured in Fig. 4(b). As described in previous sections, $V_{out}$ and $V_{in}$ will be used to estimate the deflection and the force.

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